Differential Equations — equation involving a derivative

— order highest derivative

Exact

Find all functions y that satisfy the equation

$$\frac{dy}{dx} = \sec^2 x + 2x + 5 \qquad y = \tan x + x^2 + 5x + C$$

general solution

initial value problem → y goes through pt. $(0, 6)$

particular solution

$$y = \tan x + x^2 + 5x + 6$$

Find all factors that satisfy

① $\frac{dy}{dx} = 7x^6 - 3x^2 + 5$ pt. $(1,1)$

$y = x^7 - x^3 + 5x + C$

$C = -4$

② $\frac{dy}{dx} = e^x - 6x^2$ pt. $(1,0)$

$y = e^x - 2x^3 + C$

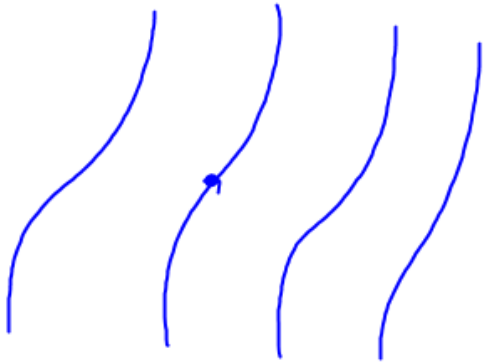
$C = 2 - e$

③ $\frac{dy}{dx} = 2x - \sec^2 x$ pt. $(0,3)$

$y = x^2 - \tan x + C$

$C = 3$

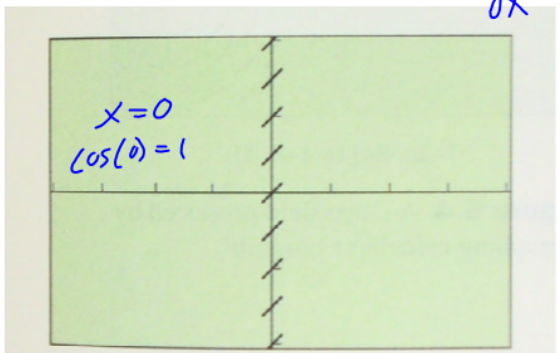
$y = x^2 - \tan x + 3, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$



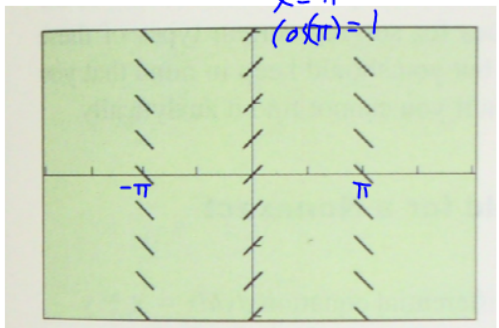
$f'(x) = e^{-x^2}$ pt $(7, 3)$

$f(x) = \int_7^x e^{-t^2}\,dt + 3$

$\frac{dy}{dx} = \cos x$



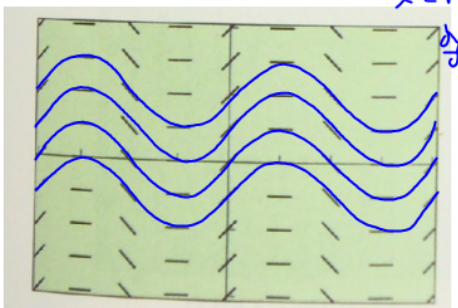
[−2π, 2π] by [−4, 4]



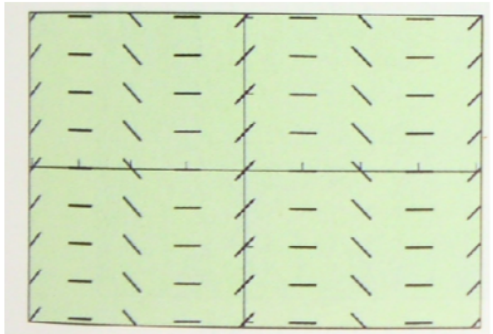
[−2π, 2π] by [−4, 4]

$x = n \cdot \frac{\pi}{2}$

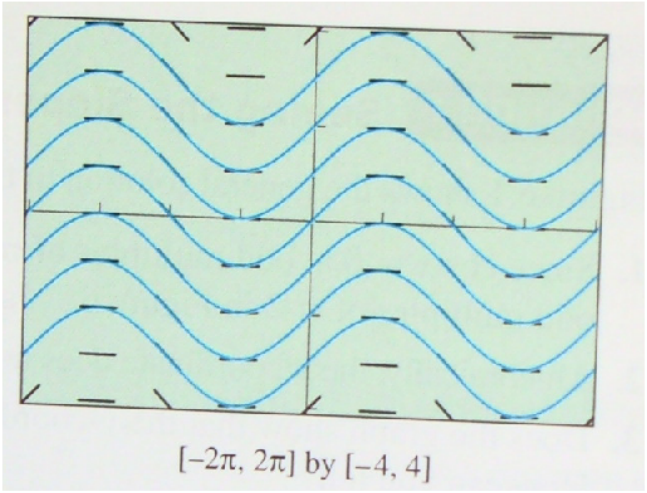
$\frac{dy}{dx} = 0$



[−2π, 2π] by [−4, 4]



[−2π, 2π] by [−4, 4]



[−2π, 2π] by [−4, 4]

$y = \sin x + C$

EXAMPLE 7 Constructing a Slope Field for a Nonexact Differential Equation

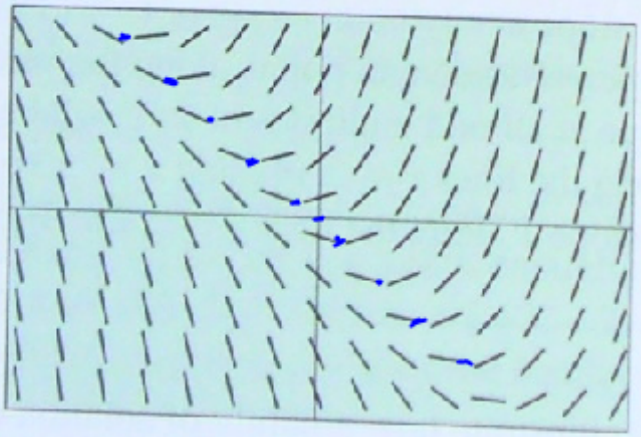
Use a calculator to construct a slope field for the differential equation $dy/dx = x + y$ and sketch a graph of the particular solution that passes through the point $(2, 0)$.

SOLUTION

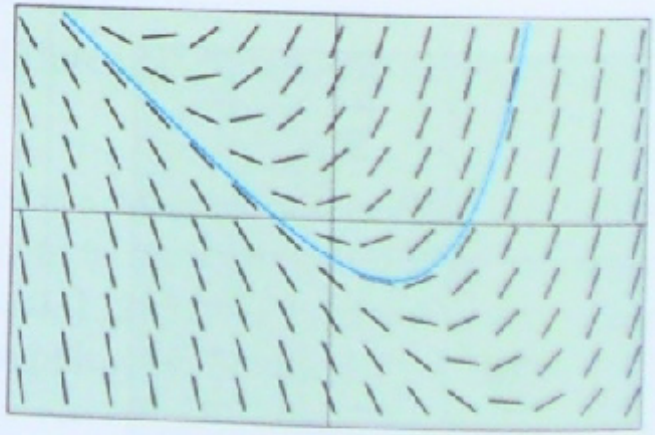
The calculator produces a graph like the one in Figure 6.5a. Notice the following properties of the graph, all of them easily predictable from the differential equation:

1. The slopes are zero along the line $x + y = 0$.
2. The slopes are -1 along the line $x + y = -1$.
3. The slopes get steeper as x increases.
4. The slopes get steeper as y increases.

The particular solution can be found by drawing a smooth curve through the point $(2, 0)$ that follows the slopes in the slope field, as shown in Figure 6.5b.



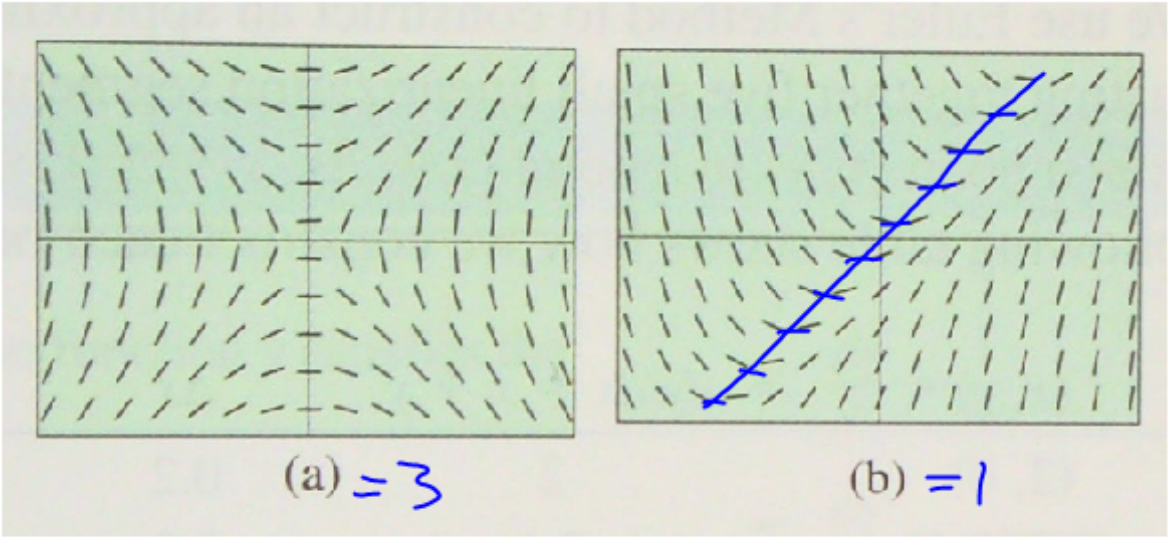
[−4.7, 4.7] by [−3.1, 3.1]



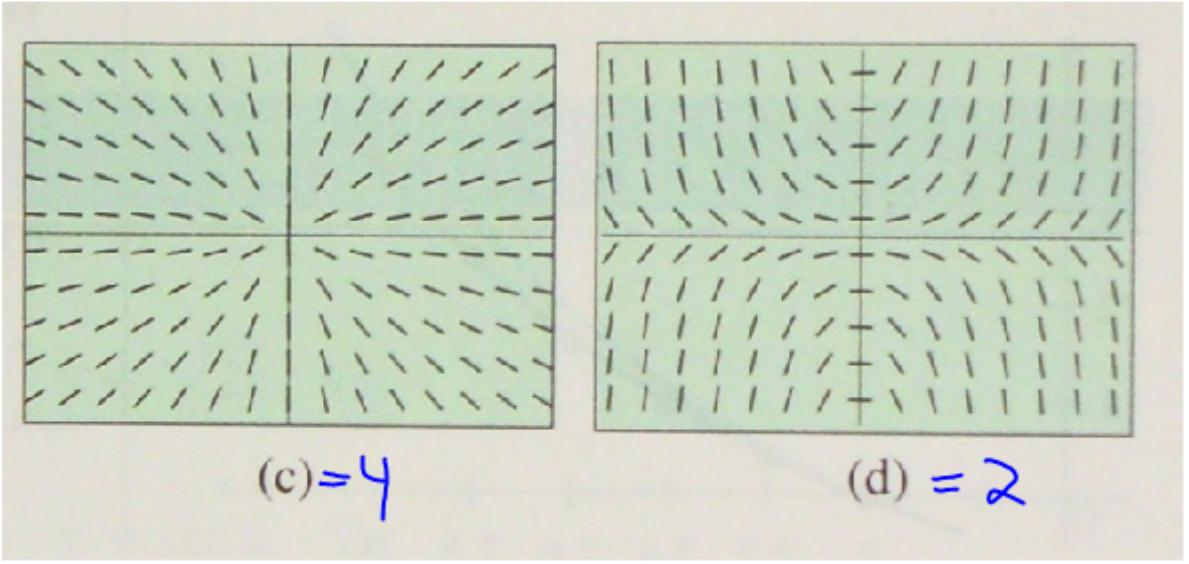
[−4.7, 4.7] by [−3.1, 3.1]

Match

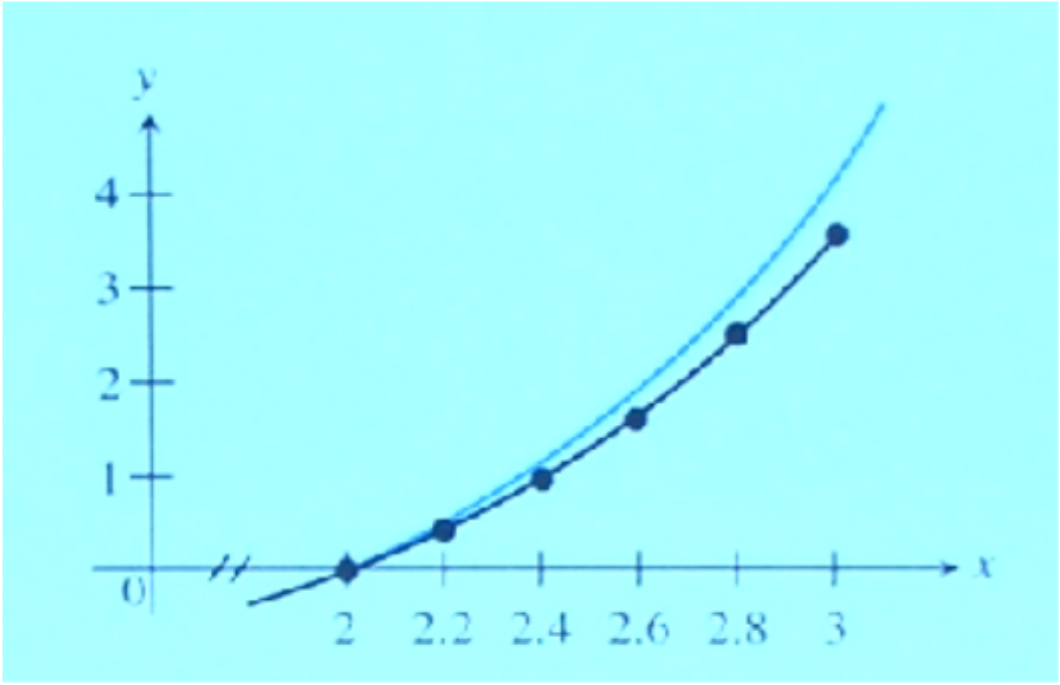
1. $\frac{dy}{dx} = x - y$ **2.** $\frac{dy}{dx} = xy$ **3.** $\frac{dy}{dx} = \frac{x}{y}$ **4.** $\frac{dy}{dx} = \frac{y}{x}$



(a) = 3 (b) = 1



(c) = 4 (d) = 2


y
4
3
2
1
0
x
2
2.2
2.4
2.6
2.8
3

Sect. 6.1

- Quick Review
- Problem set #1-24 (4 each section), 25-40, 49, 50
  (29-34 do 2)